S282 Astronomy

The geometry of the Universe

Study time: 30 minutes

Summary

This activity relates to a video sequence which provides an overview of how the geometric properties of space depend on its intrinsic curvature.

You should have read to the end of Section 5.3.4 of *An Introduction to Galaxies and Cosmology* before watching this video sequence.

Learning outcomes

- Appreciate the possibility that three-dimensional space need not have a Euclidean geometry, but may be curved in a way that is analogous to the curvature of two-dimensional surfaces.
- Understand how simple geometric tests can be used to determine the curvature of a three-dimensional space.

The activity

In Section 5.3.2 of *An Introduction to Galaxies and Cosmology* you were introduced to the idea of curved space. In particular, you saw that space can have a curvature that is zero or is negative or positive, and that these types of spaces have different geometric properties (see Figures 5.13 of *An Introduction to Galaxies and Cosmology*).

This video sequence sets out to explain the three types of geometry to which the Universe might conform. This is done by exploring the geometrical properties of three types of two-dimensional surfaces that have zero, negative and positive curvature – these are *Euclidean*, *spherical* and *hyperbolic* geometries respectively (see Table 1).

Table 1 The types of geometry described in the video sequence and their corresponding curvatures.

Geometry	Curvature
Euclidean (or 'flat')	zero
spherical	positive
hyperbolic	negative

Throughout this sequence you should bear in mind that the term 'spherical geometry' corresponds to that of space with a positive curvature, whereas 'hyperbolic geometry' refers to a space with a negative curvature. The term 'Euclidean space' is used to describe space with zero curvature – this is also referred to in Section 5.3 of *An Introduction to Galaxies and Cosmology* as a 'flat' space.

To view the video sequence:

- Start the S282 Multimedia guide and then click on The geometry of the Universe under the ‘Cosmology’ folder in the left-hand panel.
- Press the Start button to run the video sequence.

Note that although Russell Stannard concludes by saying that the geometry of the Universe is unknown, observational results do support the notion that the geometry of the Universe is Euclidean. This observational evidence is discussed in detail in Chapter 7 of *An Introduction to Galaxies and Cosmology*.

After you have watched the video sequence, read the summary provided in the ‘Notes’ below and then attempt the following questions.

Question 1

What is the definition of a ‘straight line’ on each of the three surfaces shown in the sequence: flat, spherical and saddle?

Question 2

On the surface of a sphere of radius R, the circumference of a circle is always less than $2\pi r$, where r is the radius of the circle drawn *on* the surface. What can you say about the circumference of a circle that has $r = \pi R$? What would be the circumference of the *largest* circle that could be drawn on the surface?

Notes

The list below indicates the approximate clock time at which each new topic begins.

00:00 Euclidean geometry is but one possible candidate for the geometry of the Universe.

00:15 *Euclidean (flat) geometry* is characterized by:

(i) parallel straight lines which, on being extended to infinity, remain at the same separation from each other

(ii) angles of triangles adding up to 180°

(iii) circles with a circumference $2\pi r$, where r is the radius.

01:20 *Spherical geometry* (positive curvature) is characterized by:

(i) parallel straight lines which intersect after a finite distance

(ii) angles of triangles adding up to >180°

(iii) circles with a circumference $<2\pi r$.

05:00 *Hyperbolic geometry* (negative curvature) is characterized by:

(i) parallel straight lines which progressively diverge

(ii) angles of triangles adding up to <180°

(iii) circles with a circumference $>2\pi r$.

The latter two geometries approximate to flat geometry if the geometrical figures drawn are small compared to the scale of curvature of the surface.

08:20 It was pointed out that it is not immediately obvious what kind of geometry describes the Universe and all that is in it. Measurements with triangles and circles have not been done on a scale large enough to detect which of the three geometries applies to the Universe and are, in any case, impractical.

Video credits

Presenter – Russell Stannard (The Open University)

Producer – Tony Jolly (BBC)

Answers and comments

Question 1

In all three cases the definition of a straight line is in terms of it being the line on the surface that has the shortest distance between two points.

Question 2

If we imagine the circle being centred at the North Pole, then a radius r of length πR would extend to the South Pole. The circle would thus be drawn around the South Pole, and would have a circumference of zero.

As for the largest circle that could be drawn on the surface, that would be the Equator, and hence would have a circumference $C = 2\pi R$.

(Note that this circumference is *not* the Euclidean value of $2\pi r$. In this particular case, the radius r as drawn on the surface is $r = \pi R/2$. This expression can be rearranged to give $R = 2r/\pi$, so the circumference is

$$C = 2\pi R = 4r$$

and this is clearly less than the circumference in Euclidean space, which is given by $C = 2\pi r$.)